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Branes on charged dilatonic backgrounds: self-tuning, Lorentz violations and cosmology

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Abstract: We construct an $n+q+2$ dimensional background that has dilatonic q-brane singularities and that is charged under an antisymmetric tensor field, the background spacetime being maximally symmetric in n-dimensions with constant curvature $k = 0, \pm 1$. For $k = 1$ the bulk solutions correspond to black q-branes. For $k = 0, -1$ the geometry resembles the 'white hole' region of the Reissner-Nördstrom solution with a past Cauchy horizon. The metric between the (timelike) singularity and the horizon is static whereas beyond the horizon it is cosmological. In the particular case of $q = 0$, we study the motion of a codimension one n-brane in these charged dilatonic backgrounds that interpolate between the original scalar self-tuning and the black hole geometry and provide a way to avoid the naked singularity problem and/or the need of having exotic matter on the brane. These backgrounds are asymmetrically warped and so break 4D Lorentz symmetry in a way that is safe for particle physics but may lead to faster than light propagation in the gravitational sector.

Keywords: Extra Large Dimensions, p-branes, Cosmology of Theories beyond the SM.

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Contents

1. Introduction

The brane-world scenario is providing new ideas to approach old questions such as the hierarchy problem, the cosmological constant problem and early universe cosmology [1]–[4]. The simple idea that our Universe is a brane trapped in a higher dimensional space allows for a great amount of possible realizations depending on the distribution of matter on the brane and the bulk as well as their relative dimensionality.

As usual, simplicity has been the main guideline when considering explicit realizations of the brane world. Most discussions in the current literature refer to 3-branes inside a five dimensional bulk with only gravity propagating in the bulk. Adding extra fields to solve some of the problems such as radius stabilization and to ameliorate the cosmological constant problem has also been considered [4]–[8]. But in principle there is a great degree of arbitrariness and in order to go beyond the simplest realizations we need to have a general guideline.

Clearly the best motivated brane world scenarios are those that can naturally be obtained from string theory [9]. There are actually at present few explicit realizations of quasi-realistic brane world models derived from string theory [10]. We can try to extract the general properties of those models to incorporate them on a particular framework to approach different phenomenological and cosmological properties of these scenarios.

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Following this guideline we will consider in this note a system consisting of a q-brane[1] singularity in a $d = n + q + 2$ dimensional bulk with gravity, dilaton and antisymmetric tensor fields. We find explicit solutions for the field equations for which the n-dimensional slices of the spacetime have constant curvature $n(n-1)k$, $k = 1, 0, -1$ (for a related discussion see for instance [11]). A motivation for the study of these geometries is their potential application to brane cosmology. One of the most interesting results emerging in this field has been the realization, through Birkhoff's theorem, that an additional n-brane[2] carrying ordinary matter and gravitationally coupled to the bulk, when moving in a static black hole background, actually feels a time dependent cosmology [12, 13, 14]. Therefore it is clear that the solutions presented in this work provide interesting brane cosmology backgrounds even in the regions where they are static.

The causal structure of the spacetime depends on the topology of the dimensions parallel to the external n-brane. The $k = 1$ case has been studied in the past and it corresponds to black q-branes [15], with $n + q + 2 = 10$. The global geometry consists on an asymptotically flat spacetime with a horizon and two singularities.[3] The $k = 0, -1$ solutions we find are not black q-branes. They have an interesting global structure with a horizon and only one singularity at the origin. The region between the singularity and the horizon is static, unlike the standard black hole case. Beyond the horizon it becomes time dependent, therefore corresponding to a cosmological solution for which there is no singularity at any surface of constant time.

For the $q = 0$ case, the motion of the external n-brane (a codimension one brane) can be studied easily using the usual Israel junction equations. We mostly concentrate in the regions of the bulk spacetime which are static and find the possible places where the brane can be located, cutting the space in the transverse dimension such that a $\mathbb{Z}_2$ symmetry around the brane can be imposed and a finite Planck scale in four dimensions is guaranteed.

We find that for $k = 1$ the n-brane can be located in the region of the bulk spacetime which is outside of the singularities and the horizon and therefore the extra dimension can be naturally restricted to be the region between the black hole horizon and the location of the brane, avoiding naked singularities completely. The matter in the brane is not exotic because the value of the parameter ω relating the pressure and energy density on the brane $p = \omega\rho$ lies in the physically allowed region $0 > \omega > -1$.

[1] We refrain from using the standard terminology p-brane to avoid confusion with the pressure p in the following sections.

[2] For clarity we mention the distinction between the two branes involved in our construction: the q-brane that is electrically charged under a $q+1$ form and an n-brane that is coupled to the bulk through gravity only. Until section 3, we will keep q and n general and we will restrict to $q = 0$ afterwards.

[3] Note however, that in string theory, black holes will carry several charges, and then, the extra singularity can be stabilized [17].

For $k = -1, 0$ the situation is different. The static region lies between the singularity and the horizon, therefore there are two possibilities. The space can be taken between the brane and the horizon or between the brane and the singularity. In the first case it turns out that the energy density of the brane has to be negative and in the second it is positive. In both cases ω can take physically allowed values.

We will consider in more detail the simplest system of a 3-brane in a five-dimensional bulk with dilaton and a two-index antisymmetric tensor field $B_{\mu\nu}$. Since in five-dimensions the antisymmetric tensor is dual to a vector field $\partial_\mu A_\nu = \epsilon_{\mu\nu\rho\sigma\tau}\partial^\rho B^{\sigma\tau}$, this situation is equivalent to consider a gauge field A_μ instead of $B_{\mu\nu}$. Interestingly enough the introduction of a dilaton field has been studied to ameliorate the cosmological constant problem [5, 6] and more recently a similar proposal was made regarding the introduction of a gauge field [8]. It is then natural to consider both fields together.

This generalization improves the situation with only one of the fields in several respects regarding the self-tuning of the cosmological constant.

- For vanishing gauge fields we reobtain the solution of [5, 6] as a particular case of our $k = 0$ solution. However our solutions also include possibilities not considered in [5, 6] since we consider asymmetrically warped geometries which are not 4D Poincaré invariant from the bulk point of view but are so at each location of the brane, by suitably redefining the speed of light. These cases share the property with [5, 6] that there are always naked singularities and require a fine-tuning of the dilaton couplings and the parameter ω defining the equation of state of the matter on the brane.

- For non vanishing gauge fields and $k = 0$, ω is still a constant related to the dilaton couplings. Indeed, contrary to the model [8] with a pure Reissner-Nördstrom black hole in the bulk, the presence of the scalar field requires a fine-tuning of the dilaton couplings as in the original scalar self-tuning models [5, 6]. The improvement on the self-tuning relies on the fact that, as mentioned above, the naked singularities can be avoided as long as the energy density on the brane is negative. Also ω can lie in the physically allowed region.

Since our solutions correspond to asymmetrically warped metrics they may induce explicit violations of Lorentz invariance on the brane by having gravitational waves moving faster through the bulk than through the brane. We study the variation of the speed of light with the location of the brane and find that gravitational waves move faster through the bulk only in the cases with naked singularities.

The organization of the paper is as follows. We present in section 2 the general solution of the bulk equations of motion for gravity coupled to the dilaton field and an antisymmetric tensor of rank $q+2$, for the cases $k = 0, \pm 1$, generalizing the results of [15] (see also [16]). In the next section we introduce the external n-brane with matter in a perfect fluid for the particular case of codimension one brane. We obtain the junction conditions and then obtain the geometry defined by the position of the n-brane in the bulk background. We specialise in section 4 to the five-dimensional case discussing in detail the issues of self-tuning and violation of Lorentz invariance in these backgrounds. Finally we present our conclusions and discuss some issues related to the cosmological implications of the bulk solutions.

2. General charged dilatonic q-brane background

In [12, 13] it was found that the cosmology of a brane inside a higher dimensional bulk spacetime can be studied by considering a *static* bulk geometry with the same spatial structure, i.e., a spacetime with a constant curvature, maximally symmetric subspace of the same dimensionality as the brane. For an observer on the brane, the movement of the brane in the static geometry becomes an evolving brane Universe. This is a direct consequence of Birkhoff's theorem in more than 4 dimensions. In [14] it was explicitly shown how to map the cosmological and static metrics for the case of a five-dimensional bulk without extra matter fields. Similar results hold when there are background gauge fields [8]. Notice that this is not always the case, we may have some cosmological solutions that may not be mapped to static ones in more general cases when other fields are included. In particular, in presence of a scalar field, Birkhoff's theorem does not hold [18]. Nevertheless, it is still possible to find some background solutions that remain static or, as we will see, that depend on the time coordinate only. Even if these solutions are no longer the most general solutions in the bulk, they are interesting on their own and we will focus on their study in this paper.

In this section, we generalize the electric black q-brane solution studied by Horowitz and Strominger in [15] for positive and constant spatial curvature ($k = 1$) to the case of arbitrary dimensions and arbitrary spatial curvature $k = 0, \pm 1$.[4]

We consider the coupling of gravity to a dilaton field and an antisymmetric tensor with the following action in the Einstein frame:[5]

$$S = \int d^{n+q+2}x\sqrt{g}\left(\alpha R - \lambda(\partial\phi)^2 - \eta e^{-\sigma\phi}F^2_{q+2}\right), \tag{2.1}$$

where ϕ is the dilaton field, F is a field strength $(q+2)$ form. We have left the couplings $\alpha, \lambda, \eta, \sigma$ arbitrary. Varying the action (2.1) yields the following equations

[4] The magnetically charged case is straightforward.

[5] Our conventions correspond to a mostly positive lorentzian signature $(-+\ldots+)$ and the definition of the curvature in terms of the metric is such that a Euclidean sphere has positive curvature. Bulk indices will be denoted by Greek indices $(\mu, \nu\ldots)$ and brane indices by Latin indices $(a, b\ldots)$. The Einstein tensor in the bulk will be denoted by $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu}$.

of motion:

$$
\begin{aligned}
\alpha G_{\mu\nu} &= -\frac{1}{2}\lambda(\nabla\phi)^2\, g_{\mu\nu} + \lambda\, \nabla_\mu\phi\, \nabla_\nu\phi + \\
&\qquad + \eta\, e^{-\sigma\phi}\left((q+2)F_\mu{}^{\lambda_1\ldots\lambda_{q+1}}F_{\nu\lambda_1\ldots\lambda_{q+1}} - \frac{1}{2}g_{\mu\nu}F^2\right), \qquad (2.2)\\
2\,\lambda\,\nabla^2\phi &= -\sigma\,\eta\, e^{-\sigma\phi}F^2\,, \qquad (2.3)\\
\nabla_{\mu_1}\left(e^{-\sigma\phi}F^{\mu_1\mu_2\ldots\mu_{q+2}}\right) &= 0\,. \qquad (2.4)
\end{aligned}
$$

We are looking for solution of this system for which the spacetime is a (warped) product of a q dimensional space giving the dimensionality of the q-brane singularity ($q = 0$ is simply a black hole) and an $n+2$ dimensional spacetime where the n-dimensional slices correspond to spaces of constant curvature. The electrically charged solution is given by

$$
\begin{aligned}
ds^2 &= h_-^{\frac{-4\lambda q(n-1)^2 b}{\alpha n(n+q)\Sigma^2}}\left(-h_+ h_-^{1-(n-1)b}dt^2 + h_+^{-1}h_-^{-1+b}dr^2 + r^2 h_-^b\, dx_{n,k}^2\right) + \\
&\quad + h_-^{\frac{4\lambda(n-1)^2 b}{\alpha(n+q)\Sigma^2}}\, dy_q^2\,, \qquad (2.5)\\
\phi &= \frac{(n-1)\sigma b}{\Sigma^2}\ln h_-\,, \qquad (2.6)\\
F_{try_1\ldots y_q} &= \frac{Q\epsilon_{y_1\ldots y_q}}{r^n}\,, \qquad \epsilon_{y_1\ldots y_q} = \pm 1\,, \qquad (2.7)
\end{aligned}
$$

where $dx_{n,k}^2$ is an n-dimensional spatial maximally symmetric metric of constant curvature $n(n-1)k$, $k = 0, \pm 1$.[6] The harmonic functions, $h_\pm$, depend on two constants of integration, $r_\pm$, and are given by:[7]

$$
h_+ = 1 - \left(\frac{r_+}{r}\right)^{n-1}, \qquad h_- = k - \left(\frac{r_-}{r}\right)^{n-1}. \qquad (2.8)
$$

We have defined the quantities Σ and b, in terms of the different parameters of the action, by the following expressions

$$
\begin{aligned}
\Sigma^2 &= \sigma^2 + \frac{4\lambda}{\alpha}\frac{q(n-1)^2}{n(n+q)}\,, \qquad (2.9)\\
b &= \frac{2\alpha n\Sigma^2}{(n-1)(\alpha n\Sigma^2 + 4(n-1)\lambda)}\,. \qquad (2.10)
\end{aligned}
$$

The electric charge, Q, of this background is related to the two constants of integration by

$$
Q^2 = \frac{4n(n-1)^2\alpha\,\lambda(r_+r_-)^{n-1}}{(q+2)!\,\eta\,(\alpha n\Sigma^2 + 4(n-1)\,\lambda)}\,, \qquad (2.11)
$$

[6] Notice that in general, one can add a constant to the dilaton field solution, $\phi \to \phi(r) + 2\phi_0$, by redefining the form as $F \to Fe^{\sigma\phi_0}$.

[7] Another solution has been found (in the special case of $n = 3$ and $q = 0$) in [19] and it is given by $h_+ = k - (r_+/r)^{n-1}$ and $h_- = 1 - (r_-/r)^{n-1}$.

while another combination of the constants of integration will be interpreted as the "mass" of the background. The solution with a vanishing electric charge, which will be of physical relevance concerning the problem of the cosmological constant, will be presented separately in the section 4.2.

Even if formally satisfying the equations of motion, this solution does not make sense physically in the regions where h_- becomes negative, i.e., $r < r_-$ for $k = 1$ and anywhere for $k = 0, -1$, because h_- appears in the solution with non-integer powers, contrary to the case of a simple Schwarzschild or Reissner-Nördstrom black hole. However, it is easy to construct a new solution that overcomes this problem:[8] the solution will still be given by the expressions (2.5)–(2.7) with the quantities Σ, b, Q still related to the parameters of the action by (2.9)–(2.11) but the definitions (2.8) of the functions $h_\pm$ has to be replaced by

$$h_+(r) = s(r)\left(1 - \left(\frac{r_+}{r}\right)^{n-1}\right), \qquad h_-(r) = \left|k - \left(\frac{r_-}{r}\right)^{n-1}\right|, \tag{2.12}$$

where

$$s(r) = \mathrm{sgn}\left(k - \left(\frac{r_-}{r}\right)^{n-1}\right). \tag{2.13}$$

Now, it is important to observe some of the geometrical characteristics of this solution. First of all, it is not hard to find that for all k, r_+ is always a horizon, as usual. The other properties depend on the value of the constant curvature k:

- ***k=1:*** By computing the scalar curvature, it is possible to realize that $r = r_-$ is a scalar singularity for an arbitrary value of b.[9] The background is asymptotically flat and corresponds (for $d = 10$), written in the Einstein frame, to the black q-brane solution constructed by Horowitz and Strominger [15].

- ***k=0* and *k=−1:*** It is clear from the expressions for h_- above, that $r = r_-$ is just a regular point, whereas $r = r_+$ remains a horizon. Furthermore, the coordinate r becomes timelike in the region $r > r_+$ but remains spacelike for $r < r_+$, exactly the opposite of Schwarzschild black hole. The q-dimensional singularity at $r = 0$ is then timelike. Therefore we are only interested on the static region of spacetime, and we will locate the external n-brane on the region $0 < r < r_+$. However we would like to point out here that the region $r > r_+$ is interesting per se for cosmology. In this region r becomes the time coordinate and the horizon r_+ is a past Cauchy horizon for this cosmological solution. Unlike the standard cosmological singularity $r = 0$ becomes a timelike singularity behind the horizon, resembling the 'white hole' region of the Reissner-Nördstrom solution, but having a single horizon instead of two.

[8]Notice also that it would have been possible to flip both signs in front of $(r_\pm/r)^{n-1}$ in $h_\pm$ while keeping a real electric charge, however the origin $r = 0$ would be a naked singularity.

[9]For $b = 0$, $r = r_-$ is actually another horizon as in the Reissner-Nördstrom solution.

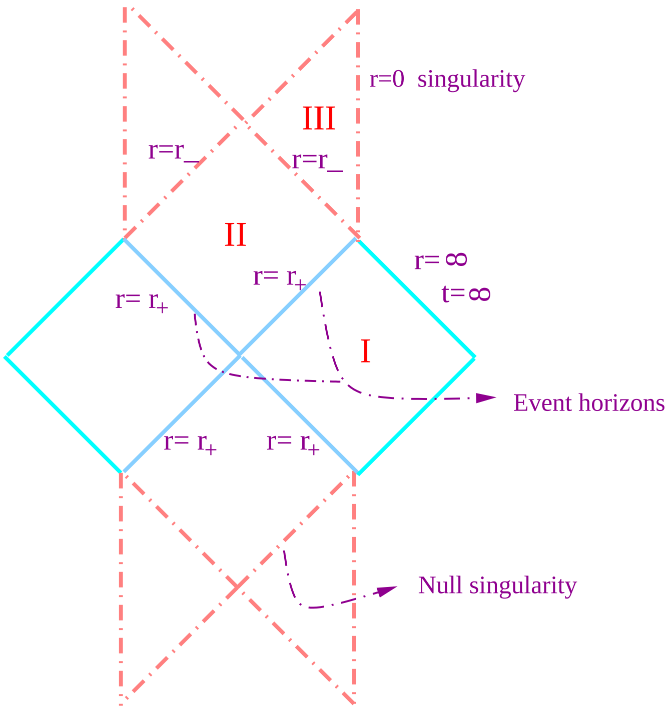

Figure 1: Penrose diagram for the $k = 1$ dilatonic Reissner-Nordström black brane.

> Unlike the Schwarzschild solution, since the singularity is timelike it may be avoided by a future directed timelike curve in the region beyond the horizon.

We show in figures 1 and 2 the corresponding Penrose diagrams for these geometries illustrating the relevant regions.

A detailed study of the external n-brane inside the cosmological regions is beyond the scope of the present article and it is left to a future publication [18]. In the next section we will restrict only to the static solutions, $r_\mp < r_\pm < r$ or $r < r_\mp < r_\pm$ for $k = 1$ and $0 < r < r_+$ for $k = 0$ and $k = -1$, keeping in mind their possible relevance for cosmology as well as the self-tuning mechanism for the cosmological constant and the possible bulk violations of the 4D Lorentz invariance.

Before finishing this section a comment is in order. In the general action above we have not included a cosmological constant term in the bulk. The reason is the following. The term would be of the form $Ve^{\nu\phi}$ with V some constant. However it has been shown [20] that for $\nu \neq 0$, and for $\nu = 0$ but $V > 0$, there are no solutions of the field equations consistent with the symmetries we imposed. The only possibility would be to have $\nu = 0$ and $V < 0$ for which some numerical solutions are known, or to restrict to the case of no dilaton couplings $b = 0$ which would reduce to the case with only gauge fields and no dilatons already considered in the literature. There still remains the possibility of a nontrivial dilaton potential $V(\phi)$ with a stationary point for which there should be solutions [20]. This possibility will not be considered here. However, it should be mentioned that, in absence of gauge fields but with a

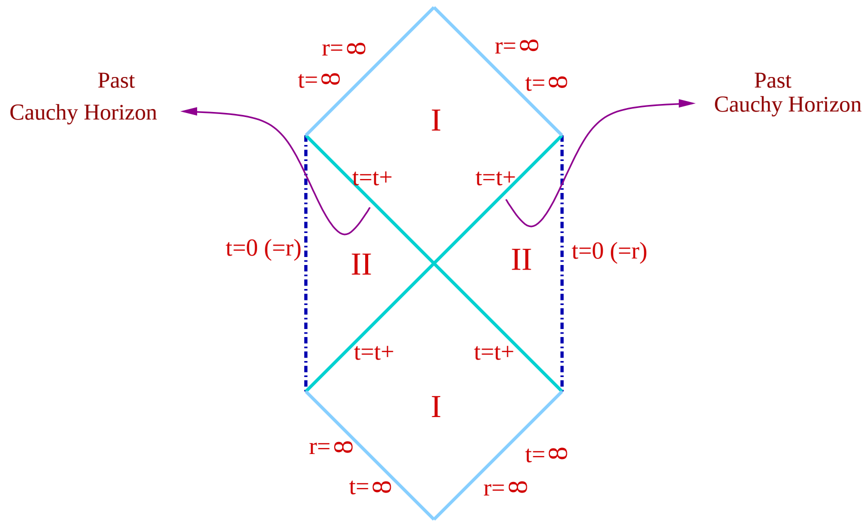

Figure 2: Penrose diagram for the $k = 0, -1$ case. This diagram is very similar to the Schwarzschild black hole (rotated by $\pi/2$), but now region I is not static, but cosmological with a past Cauchy horizon and region II is static. We put the brane in region II.

Liouville potential for the scalar field, a dilatonic domain wall ($k = 0$) solution has been constructed by Cai and Zhang in [21].

3. Codimension one brane worlds

In order to incorporate the external n-brane in the bulk geometry described before we will restrict to the case of a codimension one brane world which corresponds to the case $q = 0$, that is a point-like singularity. Therefore we are restricted to the case of an n-brane in $d = n + 2$ spacetime dimensions.

To be concrete, let us define the starting action for this model. It consists of the sum of bulk action as before (setting $q = 0$ and fixing the values of α, λ and η) plus a brane part. We have, in the Einstein frame:

$$S = \frac{1}{2\kappa^2_{n+2}} \int d^{n+2}x \sqrt{g_{n+2}} \left(R - \frac{1}{2}(\nabla\phi)^2 - 2\kappa^2_{n+2} e^{-\sigma\phi} F_{\mu\nu}F^{\mu\nu} \right) + S_{\rm br} + S_{\rm G.H.}\,, \quad (3.1)$$

with

$$S_{\rm br} = -\int d^{n+1}x \sqrt{g_{n+1}\, f^{n+1}(\phi)}\, \mathcal{L}_m(\psi, \nabla\psi, g_{ab} f(\phi))\, z(\phi)\,, \quad (3.2)$$

as before, ϕ is the dilaton field, while $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the field strength tensor of a gauge field, A_μ; $S_{G.H.}$ is the Gibbons-Hawking term and $\mathcal{L}_m$ is the lagrangian for the matter fields, ψ, living on the brane which we model as a perfect fluid; κ^2_{n+2} is the effective Newton constant in $(n + 2)$ dimensions. The matter on the brane couples to the bulk through gravity or via the dilaton only. The conformal coupling of the dilaton to matter is specified by the function $f(\phi)$ and we have also introduced a multiplicative coupling through the function $z(\phi)$. Varying the action (3.1) we will obtain an extra term coming from the brane source in the equations of motion for the metric and the dilaton. They are as follows ($r = \mathcal{R}(\tau)$ is the position of the brane):

$$\begin{aligned} G_{\mu\nu} &= -\frac{1}{4}(\nabla\phi)^2 g_{\mu\nu} + \frac{1}{2}\nabla_\mu\phi\,\nabla_\nu\phi \\ &\quad + 2\kappa_{n+2}^2 e^{-\sigma\phi}\left(2F_\mu{}^\lambda F_{\nu\lambda} - \frac{1}{2}g_{\mu\nu}F^2\right) + \kappa_{n+2}^2\sqrt{\frac{g_{n+1}}{g_{n+2}}}T_{ab}\delta_\mu^a\delta_\nu^b\delta(r-\mathcal{R}(\tau))\,, \quad (3.3) \\ \nabla^2\phi &= -2\kappa_{n+2}^2\,\sigma e^{-\sigma\phi}F^2 - 2\kappa_{n+2}^2\sqrt{\frac{g_{n+1}}{g_{n+2}}}\left((np-\rho)\frac{f'}{2f} + \omega_{\mathcal{L}}\rho\frac{z'}{z}\right)\delta(r-\mathcal{R}(\tau))\,. \quad (3.4) \end{aligned}$$

T_{ab} is the brane stress-energy tensor coupled to the induced metric on the brane and it is given by

$$T^a{}_b = \frac{2}{\sqrt{g_{n+1}}}\frac{\delta S_{\rm br}}{\delta g_{ac}}\,g_{cb} = \mathrm{diag}(-\rho, p, \ldots, p) = f^{(n+1)/2}(\phi)z(\phi)\,\mathrm{diag}(-\bar\rho, \bar p, \ldots, \bar p)\,, \tag{3.5}$$

where $\bar\rho$ and $\bar p$ would be the energy density and pressure of the brane in absence of coupling to the dilaton, while the physical energy density and pressure coupled to the induced metric in the Einstein frame are ρ and p. The action for the matter on the brane is expressed in terms of the energy density through the parameter $\omega_{\mathcal{L}}$:[10]

$$f^{(n+1)/2}(\phi)\,z(\phi)\,\mathcal{L}_m(\psi, \nabla\psi, g_{ab}f(\phi)) = -\omega_{\mathcal{L}}\,\rho\,. \tag{3.6}$$

Notice that the energy-momentum conservation reads

$$\nabla_\mu T^{\mu\nu} = 0\,; \tag{3.7}$$

where $T^{\mu\nu}$ is the total $n+2$-dimensional stress-energy tensor.

Ignoring for the moment the presence of the brane, the solution of the bulk corresponds exactly to the $q = 0$ case of the first section. This is given by

$$ds^2 = -h_+ h_-^{1-(n-1)b}dt^2 + h_+^{-1}h_-^{-1+b}dr^2 + r^2 h_-^b\,dx_{n,k}^2\,, \tag{3.8}$$

$$\phi = \frac{(n-1)b}{\sigma}\ln h_-\,, \tag{3.9}$$

$$F_{tr} = \frac{Q}{r^n}\,, \tag{3.10}$$

where now the expressions for b and Q read

$$b = \frac{2n\sigma^2}{(n-1)(n\sigma^2 + 2(n-1))}\,, \tag{3.11}$$

$$Q^2 = \frac{n(n-1)^2(r_+r_-)^{n-1}}{2\kappa_{n+2}^2(n\sigma^2 + 2(n-1))}\,. \tag{3.12}$$

[10]The value of $\omega_{\mathcal{L}}$ depends on the type of matter of the brane. Known cases correspond to $\omega_{\mathcal{L}} = -\omega$ for a time dependent scalar field [22]. There are also other cases for which $\omega_{\mathcal{L}} = 1$ [23].

And the functions $h_\pm$ are the sign amended harmonic functions (2.12). Note that depending on the value of the dilaton to the gauge field, the parameter b varies between 0 and $2/(n-1)$. Thus, even in the cases $k=0,-1$ where h_- is decreasing with r, the spatial warp factor $r^2 h_-^b$ always remains increasing: there is no bounce.

3.1 Introducing the brane

Let us now introduce a dynamical n-brane moving into the static $n+2$ bulk described in the last section. The brane will be separating two regions of the above discussed background; that is, we are gluing two slices of the metric together at the position of the brane. We will assume that our solutions possess a $\mathbb{Z}_2$ symmetry "centered" at the brane (this means essentially we change $r \to \mathcal{R}^2/r$ on the two sides of the brane and therefore identify the two sides of the spacetime in that dimension, for a detailed discussion see [8, 24]).

In order to do this, we have to satisfy the Israel junction conditions at the brane as follows [25]. Consider a general boundary X^μ, parametrized by the cosmic time τ,

$$X^\mu = (t(\tau), \mathcal{R}(\tau), x_1, \ldots, x_n)\,, \tag{3.13}$$

such that the induced metric on the boundary becomes

$$ds^2 = -d\tau^2 + a^2(\tau) dx_{n,k}^2\,, \tag{3.14}$$

with $a(\tau)$ is the scale factor on the brane, i.e.

$$a(\tau) = r\, h_-^{b/2}(r)_{|r=\mathcal{R}(\tau)} \tag{3.15}$$

and the cosmic time τ is defined such that (a dot means derivative with respect to the proper time)

$$-h_+ h_-^{1-(n-1)b} \dot{t}^2 + h_+^{-1} h_-^{b-1} \dot{\mathcal{R}}^2 = -1\,. \tag{3.16}$$

The components of the extrinsic curvature have to satisfy the junction conditions:

$$[K_{ab}]_-^+ = -\kappa_{n+2}^2 \left(T_{ab} - \frac{1}{n} g_{ab} T^c{}_c \right), \tag{3.17}$$

while the junction condition for the dilaton derives from (3.4) and reads (u is the unit normal to the brane) [26]:

$$[u.\partial\phi]_-^+ = 2\kappa_{n+2}^2 \left(-(np-\rho)\frac{f'}{2f} - \omega_{\mathcal{L}}\, \rho\, \frac{z'}{z} \right). \tag{3.18}$$

Notice that for the gauge field there are no junction conditions since it couples only to the bulk and not to the brane. With a $\mathbb{Z}_2$ symmetry between the two sides of the brane the junction equations are:

$$\rho = \mp 2n\kappa_{n+2}^{-2}\left(\frac{1}{\mathcal{R}}+\frac{b\,h'_-}{2\,h_-}\right)h_+^{1/2}h_-^{(1-b)/2}\sqrt{1+h_+^{-1}h_-^{-1+b}\dot{\mathcal{R}}^2}\,, \tag{3.19}$$

$$\begin{aligned} np+(n-1)\rho = {}& \pm 2n\kappa_{n+2}^{-2}\frac{h_+^{-1/2}h_-^{(b-1)/2}}{\sqrt{1+h_+^{-1}h_-^{-1+b}\dot{\mathcal{R}}^2}}\left(\ddot{\mathcal{R}}-\frac{(n-2)bh'_-}{2\,h_-}\dot{\mathcal{R}}^2\right. \\ & \left.+\frac{1}{2}h_-^{1-b}h'_+ + \left(\frac{1}{2}-\frac{n-1}{2}b\right)h_+h_-^{-b}h'_-\right), \end{aligned} \tag{3.20}$$

$$(np-\rho)\frac{f'}{2f}+\omega_{\mathcal{L}}\rho\frac{z'}{z} = \mp(n-1)\kappa_{n+2}^{-2}\frac{b}{\sigma}h'_-h_+^{1/2}h_-^{-(1+b)/2}\sqrt{1+h_+^{-1}h_-^{-1+b}\dot{\mathcal{R}}^2}\,. \tag{3.21}$$

Note that due to the absence of bounce in the spatial warp factor $r^2h_-^b$, there is a simple connection between the sign of ρ and the region of the spacetime cut by the $\mathbb{Z}_2$ symmetry: for positive (negative) energy density (lower (upper) signs in the previous eqs.) we keep the interior (exterior) region of the background, $r<\mathcal{R}$ ($r>\mathcal{R}$).

Even if quite messy, these junction equations have nice physical interpretation. Indeed a first combination of (3.19)–(3.21) gives us the (non)conservation equation for the energy on the brane,

$$\dot{\rho}+n(\rho+p)H = -\left((np-\rho)\frac{f'}{2f}+\omega_{\mathcal{L}}\,\rho\,\frac{z'}{z}\right)\dot{\phi}\,, \tag{3.22}$$

where H is the Hubble parameter, i.e., the time variation of $a=\mathcal{R}\,h_-^{b/2}$, the scale factor on the brane:

$$H(\tau)=\frac{\dot{a}(\tau)}{a(\tau)}=\left(\frac{1}{\mathcal{R}}+\frac{b\,h'_-}{2\,h_-}\right)\dot{\mathcal{R}}\,. \tag{3.23}$$

Another combination gives a Friedmann-type equation that relates the Hubble parameter to the energy density on the brane:

$$H^2=\frac{\kappa_{n+2}^4}{4n^2}\rho^2-\left(\frac{1}{a(\tau)}+\frac{b\,h'_-}{2\,h_-^{1+b/2}}\right)^2h_+h_-\,. \tag{3.24}$$

Finally a third combination will characterize the equation of state of the matter on the brane: $p=\omega\rho$ with

$$\omega=\left(\frac{(n-1)\,b\,h'_-}{2n\,\sigma\,h_-}\left(\frac{1}{\mathcal{R}}+\frac{b\,h'_-}{2\,h_-}\right)^{-1}-\omega_{\mathcal{L}}\frac{z'}{z}+\frac{f'}{2\,f}\right)\frac{2\,f}{n\,f'}\,. \tag{3.25}$$

In general the position of the brane will be time-dependent and so also the equation of state will be. In the interesting case of exponential couplings to the dilaton, $f=f_0\,\exp(\beta\phi)$ and $z=z_0\,\exp(\gamma\phi)$, clearly one could obtain a constant ω by tuning the parameters of the action such that $b=0$ or either by choosing a constant of integration $r_- = 0$. Surprisingly when the curvature vanishes, $k = 0$, the previous formula becomes:

$$\omega = -\frac{2(n-1)^2\, b}{n^2\, \beta\sigma\, (2-(n-1)b)} + \left(1 - \frac{2\omega_{\mathcal{L}}\gamma}{\beta}\right)\frac{1}{n}\,, \tag{3.26}$$

describing an equation of state that remains constant. Since b is related through (3.11) to the parameters that enter in the action, this expression actually fixes completely the equation of state from the original parameters of the lagrangian, and once specifying those parameters there is only one possible equation of state allowed. We may also work backwards and find which couplings in the action allow a particularly interesting equation of state (say $\omega = -1$). This is the same fine-tuning of the dilaton couplings as in the original scalar self-tuning models [5, 6]. We will come back to this issue in the next section.

It is easy to see that the equations (3.22), (3.23) and (3.24) reduce to the well known expressions for the case of no dilaton coupling ($b = 0$) and/or no gauge field coupling ($r_+ = 0$).

Before describing in detail the geometry of the brane-bulk system, let us rewrite the (non)conservation eq. (3.22) in a more usual form at least in absence of multiplicative coupling to the dilaton ($z = 1$). Indeed this equation takes a more simpler form in the Jordan frame defined by a Weyl rescaling of the metric with the conformal coupling to the dilaton:

$$d\bar{s}^2 = f(\phi)\, ds^2\,, \quad \text{i.e.,} \quad d\bar{\tau}^2 = f(\phi)\, d\tau^2 \quad \text{and} \quad \bar{a}^2 = f(\phi)\, a^2\,. \tag{3.27}$$

The energy density and pressure coupled to this metric are precisely the quantities $\bar{\rho}$ and $\bar{p}$. Then it is easy to prove that the (non)conservation equation reads:

$$\frac{d}{d\bar{\tau}}\bar{\rho} + n(\bar{\rho}+\bar{p})\bar{H} = 0 \qquad \text{with} \qquad \bar{H} = \frac{1}{\bar{a}}\frac{d\bar{a}}{d\bar{\tau}}\,. \tag{3.28}$$

In the presence of a multiplicative coupling, $z \neq 1$, the right hand side would not vanish, but as soon as $z = 1$, our model is simply a Brans-Dicke theory with localized matter on a brane and it is well known that the usual conservation equation holds in the Jordan frame.

3.2 Geometry of the brane-bulk system

Now we have to decide which part of the space-time to keep due to the $\mathbb{Z}_2$ symmetry we are requiring in order to guarantee a compact extra dimension and then finite four-dimensional Planck scale. This will be defined by the normal vector that we choose in the calculation of the extrinsic curvature and it is reflected in the sign of the energy density, as we have already explained from the junction conditions. If the normal vector points inwards, the energy density is positive whereas for outwards normal vector it is negative. So if $\mathcal{R}(\tau)$ represents the location of the brane, we can glue two interior, $r < \mathcal{R}$ (exterior, $r > \mathcal{R}$), regions by taking a positive (negative) energy density.

There will be three (two) zones (see figures 1 and 2), defined by the black hole/0-singularity geometry, where the brane can be located for $k = 1$ ($k = 0, -1$), in either case, we take two exterior or interior regions. Let us analyze this point in detail.

- ***k=1* case**

(I) $r_\mp < r_\pm < \mathcal{R}(\tau)$

a) $\rho < 0$ Glue two exterior regions with no horizon or singularities in it. So extra space dimension is infinite.

b) $\rho > 0$ Glue two interior regions which contain the two singularities $r = 0$, r_-, protected by the horizon at r_+ if $r_+ > r_-$ in which case one can naturally cut the space at the horizon and avoid in a natural way the singularities with a positive energy density on the brane. However, if $r_+ < r_-$ from the brane point of view, one will see the singularity r_- and not the horizon hidden behind it.

(II) $r_- < \mathcal{R}(\tau) < r_+$

a) $\rho < 0$ Glue two exterior regions which include the horizon at r_+ and no singularities at all.

b) $\rho > 0$ Glue two interior regions which contain the two naked singularities $r = 0$, r_-.

(II') $r_+ < \mathcal{R}(\tau) < r_-$

a) $\rho < 0$ Glue two exterior regions which include the the naked singularity at r_-.

b) $\rho > 0$ Glue two interior regions which contain the horizon at r_+.

Notice that in both cases (II) and (II') r is a timelike coordinate in the interval $[r_\mp, r_\pm]$ and so the space-time there is cosmological. The singularity at r_- is a null-singularity.

(III) $\mathcal{R}(\tau) < r_\mp < r_\pm$

a) $\rho < 0$ The metric becomes again static and one can glue two exterior regions with the singularity at r_- and the horizon at r_+ that shields the singularity as long as $r_+ < r_-$. However, if $r_- < r_+$ then from the point of view of the brane, one will see a naked singularity and not the horizon, located behind the singularity! In both situations, one can then cut the space at the singularity or the horizon to get a finite extra dimension.

b) $\rho > 0$ Glue two interior regions which contain the naked singularity at $r = 0$. One can cut the space at the singularity there.

In some cases we can see that it is possible to cut the space in such a way that naked singularities are avoided, thus getting rid of the problems described for example in [5, 27]. For instance we can choose naturally the brane as described in (I)b: we then require $r_- < r_+ < \mathcal{R}(\tau)$ and a positive energy density.

- ***k=0* and *k=−1* cases**

(I) $r_+ < \mathcal{R}(\tau)$

a) $\rho < 0$ The metric in this region has become time-dependent and is therefore cosmological. One can still glue two exterior regions with no horizon or singularities in it.

b) $\rho > 0$ Glue two interior regions which contain the singularity at $r = 0$ protected by the horizon at r_+. The bulk metric is again cosmological.

(II) $\mathcal{R}(\tau) < r_+$

a) $\rho < 0$ The metric becomes again static and one can glue two exterior regions keeping the horizon at r_+ and then cutting there the space to obtain a finite extra dimension.

b) $\rho > 0$ Glue two interior static regions which contain the naked singularity at $r = 0$. One can cut the space at the singularity getting a finite extra dimension.

There are two possibilities to obtain a finite extradimension and thus a finite Planck scale on the brane: putting the brane at $\mathcal{R} \leq r_+$ and, taking a negative energy density and avoiding the singularity at $r = 0$ as described in (II)a; or taking a positive energy density but dealing again with a naked singularity as described in (II)b. We will see later, that any of these choices is perfectly consistent with the positivity theorems of the energy density on our brane.

4. The 5 dimensional model

In this section we would like to describe some interesting features of the model presented in the case when we have a 3-brane representing our world. In order to do that, we just have to take $n = 3$ in all the relations of the last section.

Besides simplicity and the obvious relevance of 3-branes, this will allow us to make contact with the solutions discussed in the literature for 5 dimensions. We will consider the various limits of our solution to reobtain the different systems already considered in the literature for 5 dimensions where only gauge fields or only dilaton fields were considered.

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(i) In the limits $r_+ = b = 0$ or $r_- = b = 0$ both the gauge and dilaton fields vanish and we end up with the solution given in [14] with vanishing bulk cosmological constant which, for $k = 1$ corresponds to the usual Schwarzschild black hole solution.

(ii) In the limit $b = 0$, one obtains a solution with a vanishing dilaton field, that corresponds to the model found in [8] again without bulk cosmological constant.

(iii) The case $r_+ = 0$, $b = 2/3$, $k = 0$ corresponds to the usual scalar self-tuning geometry, as we will see below.

4.1 Static 3-brane universe

We will discuss now in detail the implications of the junction conditions for matter on the 3-brane, determined by the energy density and the pressure. First we will examine under which conditions the brane remains static in the bulk: $\mathcal{R}(\tau) = R_0$. The static brane solution is of particular relevance in the case of a vanishing curvature, $k = 0$, since the induced metric on the brane is then 4D Poincaré invariant.

We will restrict to exponential couplings between the brane and the dilaton:

$$f(\phi) = f_0 \, e^{\beta\phi} \, , \qquad z(\phi) = z_0 \, e^{\gamma\phi} \, . \tag{4.1}$$

Thus according to the junction equation (3.26), the equation of state for a static brane corresponds to ω =const.

Remember that for $k = 0$, the only region where the brane can remain static is $r < r_+$. In order to avoid the singularity problem we will deal with negative energy density and glue two exterior regions ($r > R_0$).

For a static brane, the junction equations (3.19)–(3.21) simplify ($\dot{\mathcal{R}} = \ddot{\mathcal{R}} = 0$), leaving us with

$$\frac{1}{6}\, \kappa_5^2 \, \rho = -\frac{(1-b)}{R_0} \left(\frac{r_-}{R_0}\right)^{1-b} \sqrt{\left(\frac{r_+}{R_0}\right)^2 - 1} \, , \tag{4.2}$$

$$\frac{1}{6}\, \kappa_5^2 \, (2 + 3\omega)\rho = \frac{1}{R_0} \left(2b - 1 - \frac{r_+^2}{r_+^2 - R_0^2}\right) \left(\frac{r_-}{R_0}\right)^{1-b} \sqrt{\left(\frac{r_+}{R_0}\right)^2 - 1} \, , \tag{4.3}$$

$$\omega = \frac{1}{3} - \frac{2\omega_{\mathcal{L}}\, \gamma}{3\beta} - \frac{4\, b}{9\beta\, (1-b)\sigma} \, . \tag{4.4}$$

At this point, it should be noticed that the interpretation of one combination of the jump equations as the conservation equation is erroneous for a static brane and even if the conservation equation is trivially satisfied, we really have three independent junction conditions. A priori we have also three constants of integration R_0, r_+ and r_-. We should mention that, the combination (4.4) of the jump equations involves only parameters that appears in the action and therefore requires a kind of fine-tuning; we will discuss this issue in the next subsection. The two other equations fix the two constants of integration r_+ and r_-:

$$r_+ = R_0\sqrt{1+\frac{1}{3(1-b)\omega}}\,, \qquad r_- = R_0\left(-\frac{1}{6}\kappa_5^2\,\rho R_0\sqrt{\frac{3\omega}{(1-b)}}\right)^{1/(1-b)}\,. \tag{4.5}$$

And the consistency of the solution that requires $r_+ > R_0$ translates into

$$0 \le \omega\,. \tag{4.6}$$

Notice that since the energy density on the brane is negative, the weak energy conditions are violated anyway; in particular even if matter on the brane can have a non exotic equation of state ($0 < \omega < 1$), still $p + \rho < 0$.[11]

Even if of less physical relevance, a similar study can be conducted for non vanishing curvature, $k = \pm 1$. Let us consider, for instance, $k = +1$. The brane, with positive energy density, will be located in the region $r_\mp < r_\pm < R_0$. It is possible to see that in this case it is not necessary to require fine-tuning between the parameters of the action, and a possible expression for ω is the following:

$$\omega = -\frac{1}{3}\frac{R_0^2 - r_-^2}{R_0^2-(1-b)r_-^2}\left(2+\frac{r_-^2}{R_0^2-r_-^2}+\frac{r_+^2}{R_0^2-r_+^2}\right). \tag{4.7}$$

The formula (4.7) shows that ω is always less than zero; the physical requirement $\omega \ge -1$, moreover, is satisfied when the following condition holds

$$(1-3b)\frac{r_-^2}{R_0^2-r_-^2} \le 1 - \frac{r_+^2}{R_0^2-r_+^2}\,. \tag{4.8}$$

This last expression will be interesting later on. Notice that as long as (4.8) is satisfied, the weak energy conditions hold.

4.2 Self-tuning solutions

Let us discuss the issue of the self-tuning of the cosmological constant that has attracted some interest recently [5, 6]. The main idea behind this proposal is that the field equations in 5-dimensions with a dilaton field allow for a solution which is 4D Poincaré invariant whatever the vacuum energy on the brane is. Furthermore there are not other maximally symmetric solutions and the brane is such that any corrections to the cosmological constant coming from matter loops on the brane can be absorbed into a shift of the dilaton field, allowing the 'self-tuning' of the cosmological constant. The main problem of this proposal [5, 27] is the existence of naked singularities in the extra dimensions which cannot be avoided [7].

[11] We thank J. Cline and H. Firouzjahi for discussion on this point.

A similar proposal has been made regarding the inclusion of gauge fields instead of the dilaton [8]. Again the cosmological constant can be self-tuned by adjusting the values of the charge and mass of the corresponding AdS black hole solution. However the geometry of the brane-bulk system is such that the singularity of the black hole can be shielded by a horizon only for a brane with an exotic equation of state $\omega < -1$. In this subsection we would like to study the possibility of obtaining a self-tuning brane in our dilatonic backgrounds.

First we have to address an important issue regarding equation (3.26). When the gauge fields are nonvanishing, we proved that ω was completely determined by the parameters of the lagrangian, because the constant b was determined by σ, the coupling of the dilaton to the gauge field, as in equation (3.11). This means that in order to get a specific equation of state we would need to tune the parameters of the action (γ, β, σ). When the gauge fields vanish i.e., $Q = 0$, b is an arbitrary constant of integration (only limited by $0 \leq b \leq 1$) independent of the parameters of the action, however, since $r_+ = 0$, the junction conditions completely determine b in terms of ω and once again (3.26) requires a fine-tuning between the equation of state of the matter on the brane and the dilaton couplings. This is the same fine-tuning problem as in the original scalar self-tuning models [5, 6].

Actually the uncharged dilatonic background solution is given by

$$\begin{aligned} ds^2 &= -h_+ h_-^{1-2b} dt^2 + h_+^{-1} h_-^{-1+b} dr^2 + r^2 h_-^b \, dx_{3,k}^2 \,, \\ \phi &= \pm\sqrt{3b(1-b)} \, \ln h_- \,, \end{aligned} \tag{4.9}$$

where the functions $h_\pm$ are now given by

$$h_+ = \operatorname{sgn} h \,, \qquad h_- = |h| \,, \qquad h = k - \frac{l}{r^2} \,, \qquad k = 0, \pm 1 \,. \tag{4.10}$$

and l is an integration constant which can be positive or negative since there is no reality constraint for the electric charge anymore. The jump equations on a brane with positive energy density, for $k = 0$, now read:

$$\begin{aligned} \kappa_5^2 \rho &= 6(1-b) \frac{1}{R_0} \left(\frac{-l}{R_0} \right)^{1-b} \,, \\ b &= \frac{1 + 3\omega}{3\omega} \,, \\ (3\omega - 1) \frac{f'}{2f} + \omega_{\mathcal{L}} \frac{z'}{z} &= \mp \sqrt{\frac{b}{3(1-b)}} \,. \end{aligned} \tag{4.11}$$

A sufficient condition for r to be spacelike is the choice $k = 0$ and $l < 0$; for simplicity we will take $l = -1$. Then we can realize that the full metric (4.9) will be 4D Poincaré invariant under the condition that the spatial and temporal warp factors have the same dependence on r:

$$r^{-2(1-2b)} = r^2 r^{-2b} \,, \quad \text{i.e.,} \quad b = \frac{2}{3} \,. \tag{4.12}$$

So, we are left with the bulk metric:

$$ds^2 = \sqrt{1 - \left|\frac{y}{y_0}\right|}\left(-dt^2 + dx^2_{3,k=0}\right) + dy^2\,, \tag{4.13}$$

where the new coordinate y is related to r by: $r^{1/3}dr = dy$. This is just the bulk metric of the original scalar self-tuning model [5, 6], corresponding to the solution II of [6]. It can easily be verified that for $\beta = 1/\sqrt{6}$ and $\gamma = 0$ we get $\omega = -1$ which is the equation of state used in [6].[12]

We can actually go beyond the results in [5, 6] by noting that the other solutions with different values of b will also allow for a 3-brane with a Poincaré invariant induced metric whatever the value of its vacuum energy is. These other solutions will break the 4D Poincaré invariance in the bulk, which is still safe for gauge interactions of the Standard Model living on the brane, and may lead to interesting observational consequences in gravitational waves experiments as we will discuss in the next section. Nevertheless there is no way to overcome the problem of the naked singularity in these cases since none of the self-tuning solutions with vanishing gauge field involves a horizon.

In the case of non vanishing gauge fields, even if the value of ω is again completely determined by the parameters of the lagrangian (β, γ, σ), the global geometry is such that in the case of negative energy density on the brane, there are no naked singularities present. Also ω can be in the physically allowed range, which is an improvement on the cases with only dilaton or only gauge fields.

It is easy to verify that the other maximally symmetric four-dimensional spaces, i.e. the de Sitter and anti-de Sitter geometries, cannot be obtained from our solutions.

4.3 Violation of 4D lorentz invariance

It has recently been emphasized [8, 29] that brane world models can have an interesting effect regarding the speed of propagation of light and gravitational signals. Indeed the general solution (2.5) breaks the 4D Poincaré invariance in the bulk and as a consequence the speed of propagation of electromagnetic signals parallel to the brane depends on the location of the brane. It is then a priori possible to foresee signals that travel faster through the bulk than on the brane. It has been argued for a long time [28] that faster than light propagation and/or variation of the speed of light can solve many of the cosmological puzzles (horizon problem, cosmological constant problem, etc). These ideas witness today a renewed interest in the context of brane world models [29].

The violation of 4D Lorentz invariance was extensively discussed in [8], where the authors shown the correctness of the intuitive idea for which if one has a decreasing speed of gravitational waves moving away from the brane, then the brane Lorentz

[12] To make the comparison with [5, 6] equations we have to redefine the dilaton field appropriately.

invariance can be recovered, in the sense that the gravitational waves prefer to move on the brane, due to the Fermat's principle.

We can now calculate in which cases we can have a negative derivative for the speed of gravitational waves, i.e., a decreasing speed of light away of the brane such that according to Fermat's theorem the gravitational waves will actually propagate on the brane rather than through the bulk.

Let us first examine the case of a static brane with a 4D Poincaré invariant induced metric, which requires a vanishing curvature $k = 0$. From the expression (3.10) of the metric, we deduce the local speed of propagation of gravitational waves in a direction parallel to the brane:

$$c^2_{\mathrm{grav}}(r) = \left(\frac{r_+^2}{r^2} - 1\right)\left(\frac{1}{r^2}\right)^{(2-3b)} r_-^{2(1-3b)} . \tag{4.14}$$

This expression is of course valid only in the region where r is a spacelike coordinate. It is easy to see that, since $0 < b < 1$, the local speed of propagation $c^2_{\mathrm{grav}}(r)$ is always a decreasing function of r in the region $0 < r < r_+$, this means that this local speed of propagation will be either decreasing or increasing away from the brane in R_0 depending on the sign of the brane energy density (see figure 3):

- Positive energy density. We keep the interior region ($r < R_0$) and thus the speed of propagation is increasing away from the brane and the gravitational waves will prefer to propagate through the bulk. Note that in this case the naked singularity at $r = 0$ is not shielded by a horizon.

- Negative energy density. We keep now the exterior region ($R_0 < r < r_+$), therefore in this case the gravitational waves will prefer to travel through the brane instead of the bulk and there will be no evidence of Lorentz violation.

We can then conclude that Lorentz violation would be manifest only in the case with naked singularity, similar to the situation found in [8].

We can also consider the case $k = 1$ (although it is less interesting since the induced metric on the brane is not Poincaré invariant). In this case the conclusion regarding the increase or decrease of the speed of gravitational waves is different. The speed of the gravitational waves as a function of the position in the bulk is given by the following expression

$$c^2_{\mathrm{grav}}(r) = \left(1 - \frac{r_+^2}{r^2}\right)\left(1 - \frac{r_-^2}{r^2}\right)^{1-3b} r^{-2} , \tag{4.15}$$

and we have a brane with positive energy density located in the region $r_+ < R_0$.

The condition of decreasing speed of gravitational waves away from the brane is satisfied when

$$R_0^2 + 3(1-b)\frac{r_-^2 r_+^2}{R_0^2} \leq 2r_+^2 + (2-3b)r_-^2 . \tag{4.16}$$

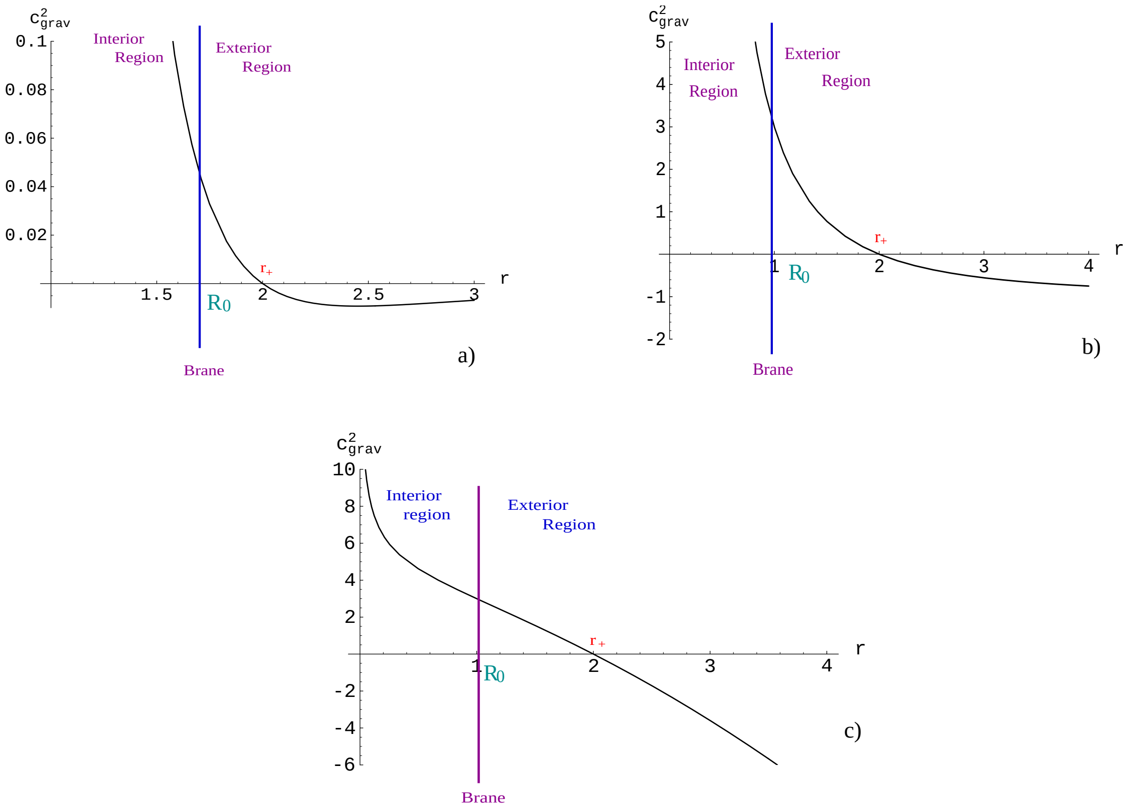

Figure 3: Speed of gravitational waves as a function of the extradimension r for $k = 0$ and a) $b = 0.001$, b) $b = 2/3$, c) $b = 0.95$. The brane is always located inside the horizon, sited at $r = r_+$. We took $r_- = 1$, $r_+ = 2$.

The behaviour of the speed of gravitational waves is clear from the plots in figure 4. It is very interesting to note that this condition is exactly the same as (4.8), but with the opposite sign. The conclusion is that, with $k = 1$, to have a decreasing speed of light moving away from the brane one needs to consider an exotic form of matter living on the brane, with $w \leq -1$. Vice-versa, if the matter on the brane has a standard equation of state, one must take into account possible non negligible effects of changes in the speed of gravitational waves in the visible world.

5. Conclusions and comments

We have started a general treatment of higher dimensional singular geometries for which external branes of arbitrary dimension can be incorporated. There are two kinds of branes in our solutions that should not be confused. First are the q dimensional bulk singularities that in the $k = 1$ case are black branes and in general are 'q-brane singularities'. The second brane is an external n-brane that is added given that the geometry factorizes (with a warp factor) between the q dimensional part and the $n + 2$ dimensional for which the n dimensional hypersurfaces are maximally

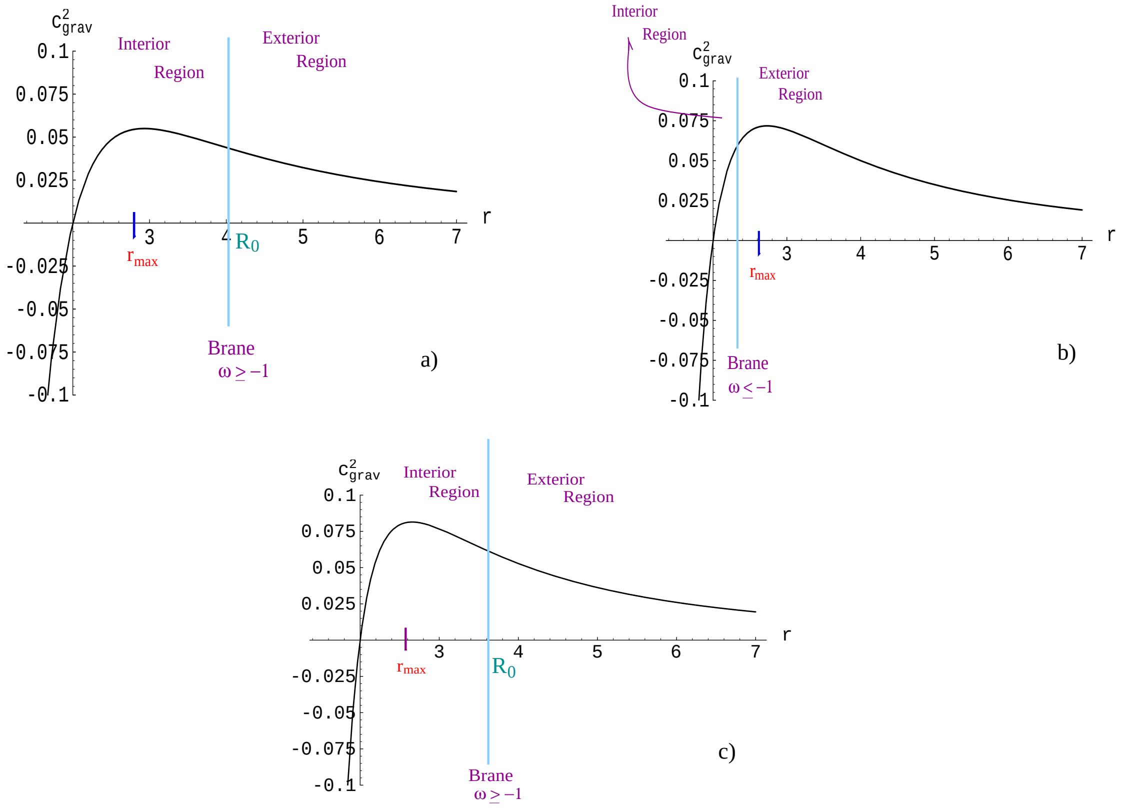

Figure 4: Speed of gravitational waves as a function of the extradimension r for $k = 1$ and a) $b = 0.001$, b) $b = 2/3$, c) $b = 0.95$. Here is clear that for a non exotic matter living on the brane we need to locate the brane to the right hand side of the maximum, having then Lorentz violation effects (we always keep the interior region: $r < R_0$). We took $r_- = 1$, $r_+ = 2$.

symmetric with constant curvature. We may see this, in the case $k = 1$, as a brane in a background defined by a second, orthogonal, brane. In the extremal case ($r_- = r_+$) the bulk solution is supersymmetric and there may be an interesting reformulation of this configuration in terms of the AdS/CFT correspondence. For $k = 0, -1$ there is no analogue to an extremal case: there are no time-like Killing vectors and there does not seem to be a supersymmetric limit.

Besides their intrinsic relevance, these configurations can be seen as the starting point of brane cosmology in the higher dimensional space. We have found several interesting properties of our solutions which are novel. First the $k = 0, -1$ bulk solutions are to the best of our knowledge new[13] and have very interesting geometrical structure having a natural cosmological interpretation but with a time-like singularity and a Cauchy horizon. Similar geometries have appeared in the past. The Penrose diagram of the Taub-NUT solution has the same signature change, as we have, in the Cauchy horizon, but in that case there is another horizon rather

[13] See however [19, 30] for related solutions.

than a singularity which allows the infinite extension of the diagram [31]. A similar behaviour appears in the study of tilted Bianchi cosmologies. In these cases the Cauchy surface corresponds actually to a non-scalar singularity usually referred to as a 'whimper' or intermediate singularity, where the metric changes signature [32]. The origin of this intermediate singularity is the fact that the fluid matter flows in a direction which is not perpendicular to the homogeneity surfaces. In our case there is no fluid in the bulk and the intermediate surface is not a singularity but a horizon. It is interesting to notice that if an observer in the region beyond the horizon extrapolates back in time she/he never finds a big-bang singularity but the horizon.

Even though a detailed study of the cosmology of these configurations is beyond our scope, we would like to point out the following interesting properties. For $k = 0$ in the 5d bulk cosmological region, at late times ($t \gg r_+$ so that $h_+ \sim -1$) the three standard spatial dimensions always expand while the fifth has different behavior depending on the values of b, similar to the Kasner solution: For $2/3 < b < 1$ all 4 spatial dimensions expand, the 5th faster than the rest. For the critical case $b = 2/3$ the four spatial dimensions expand at the same rate. In the interval $1/2 < b < 2/3$ three dimensions expand faster than the 5th, whereas for $b = 1/2$ the 5th dimension is static and the others expand. Finally for $0 < b < 1/2$ the 5th dimension contracts and the others expand. All of this in the Einstein frame.

Second our solutions provide new examples of violation of 4D Lorentz invariance that may have interesting consequences regarding gravitational waves experiments and cosmology. Finally the introduction of the n brane on the bulk static regions can be done in such a way that the singularities can be avoided. A substantial improvement on the situations with only gauge fields or only dilaton regarding the self tuning mechanism for the cosmological constant was achieved. It remains to be seen to what extent this is real progress in the approach to the cosmological constant problem and to what extent it is realistic to have a negative energy density brane world.

Many things remain to be explored in this subject. The possibility of a nontrivial dilaton potential with a stationary point is an open question. Also the incorporation of further antisymmetric tensors of different rank as well as the consideration of codimension larger than one brane worlds and the addition of more than one external branes are left to the future. Probably a more direct extension of the present work is the actual use of these geometries to a detailed cosmological study of this class of brane worlds.

We have made a step forward in studying the structure of a general class of brane models with properties close to what we natural expect in string theory. Their interesting properties makes them worth of further investigation, especially in the context of brane cosmology. We hope to report on progress in this direction in the near future.

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